Nonlinear Considerations in Energy Harvesting

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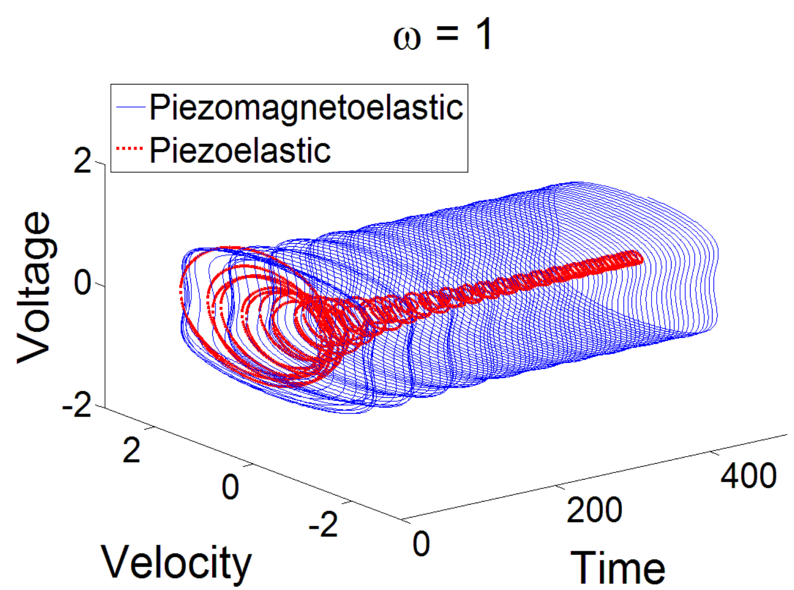
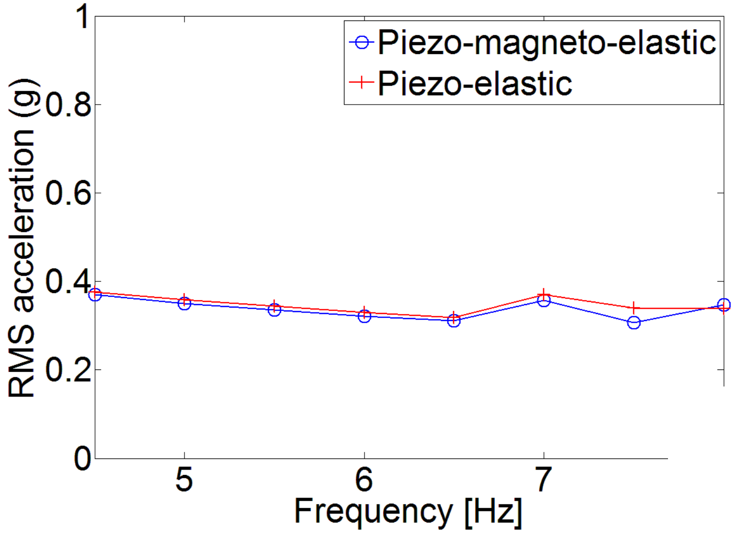


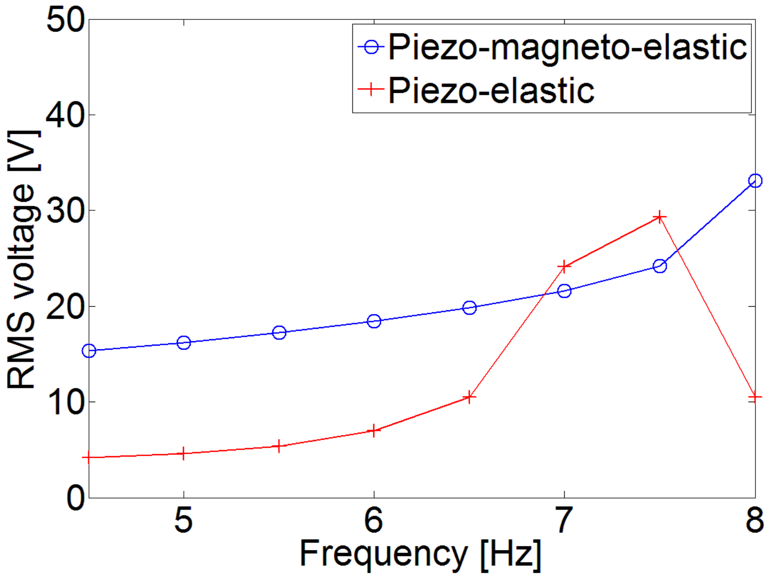
*Woodruff School of Mechanical Engineering, Georgia Tech

Two key issues in energy harvesting can be solved by introducing nonlinear effects

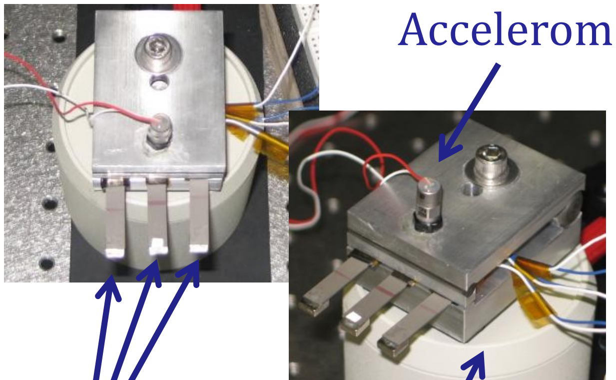


For identical inputs to a linear system and a nonlinear system ….

… a drastically improved voltage results which is broadband and of larger magnitude.



There are many conventional ways of making broadband energy harvesters…

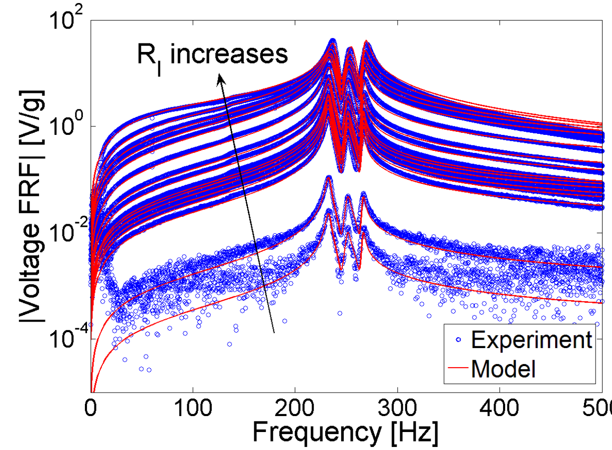


Accelerometer

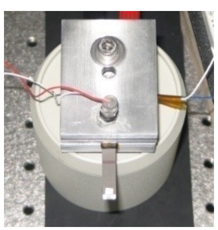
Piezoelectric bimorphs

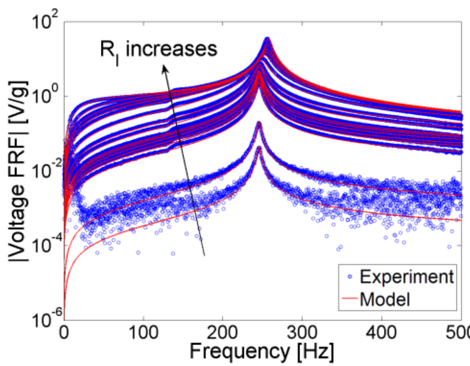
Electromagnetic shaker

Voltage FRFs (broadband configuration)

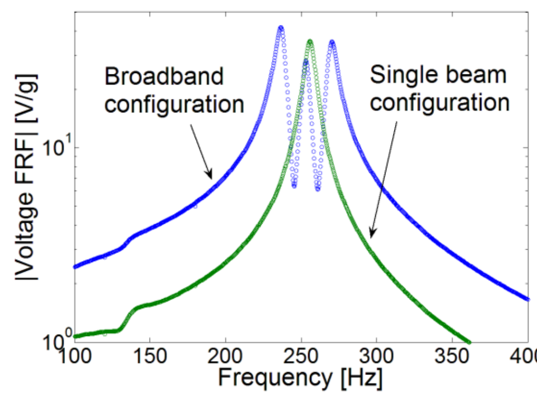


Voltage FRFs (single cantilever)





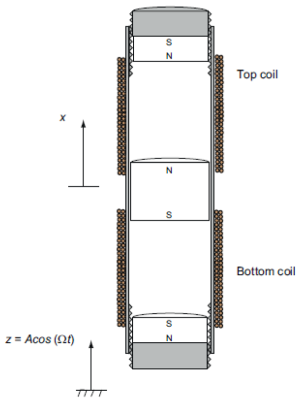
Comparison of the voltage FRFs

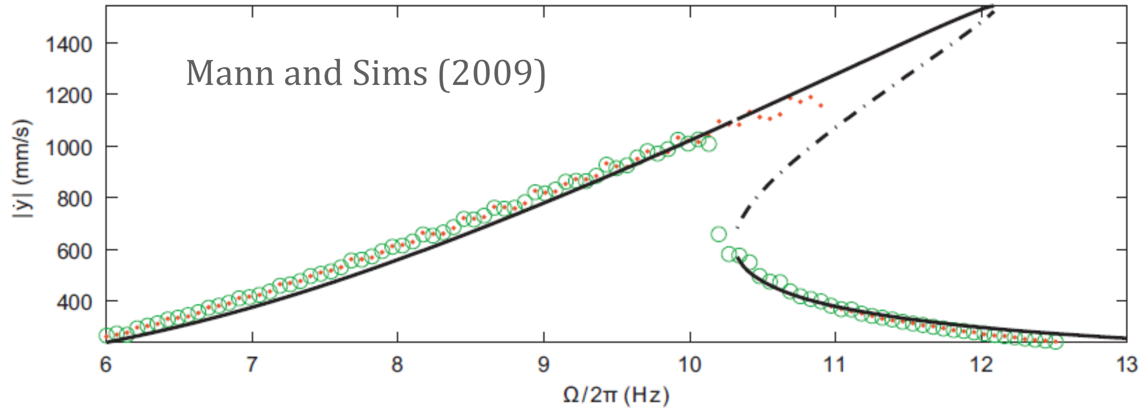


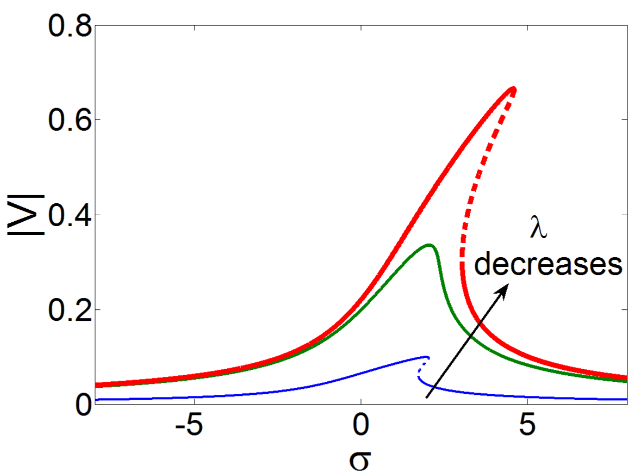
Not an outstanding design in terms of the power density…

Others use electromagnetic transduction (Beeby, et al)

Hardening stiffness of the monostable Duffing oscillator has been investigated by others to increase the bandwidth of operation.







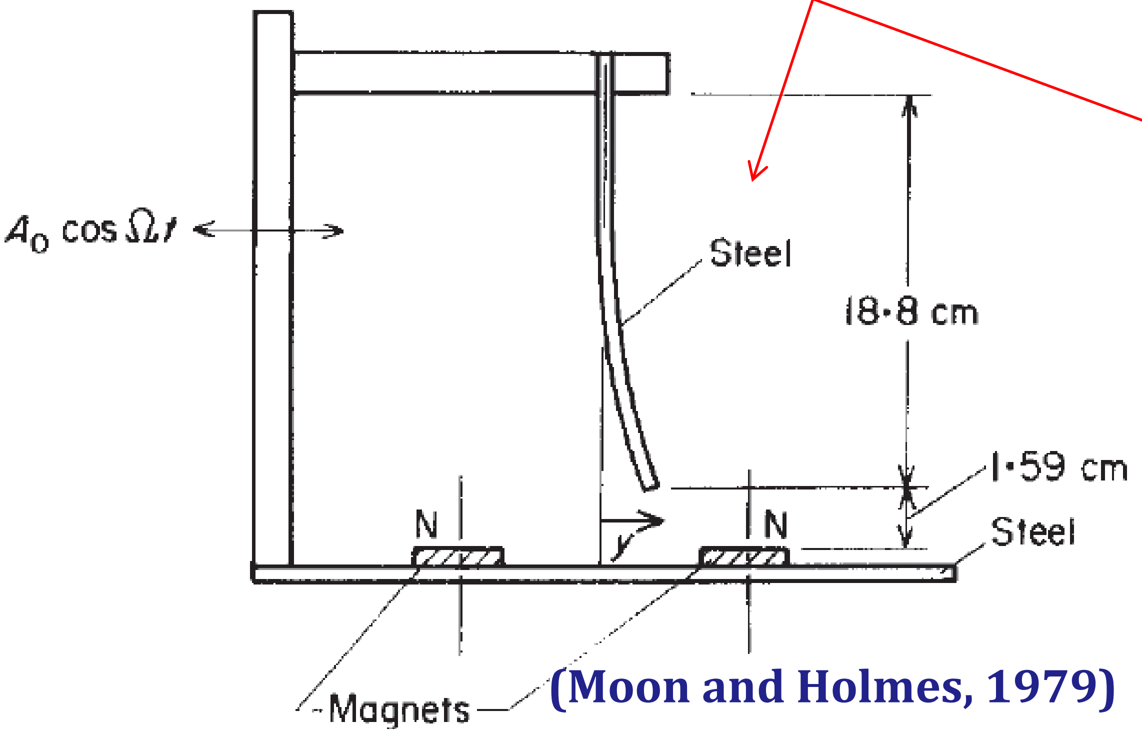
Piezoelectric energy harvester with cubic stiffness

$$\ddot{x} + 2\varepsilon\mu\omega_n\dot{x} + \omega_n^2 x + \varepsilon\alpha x^3 - \varepsilon\chi v = \varepsilon f \cos\omega t$$

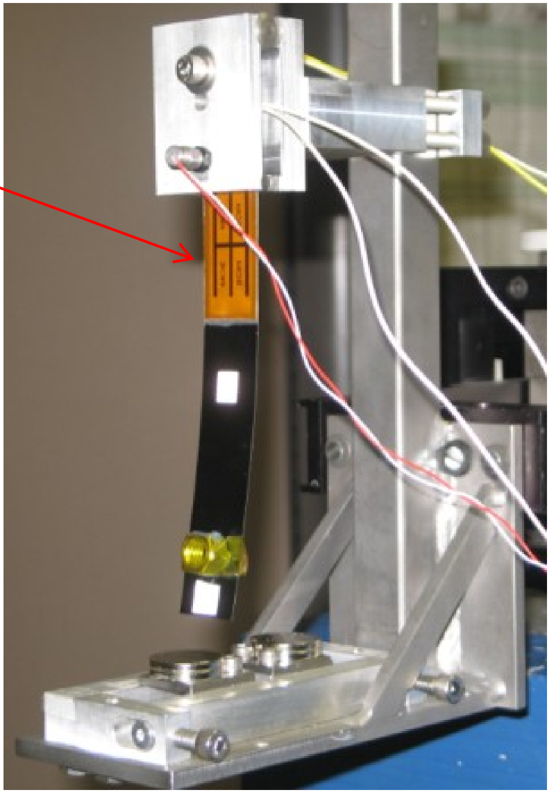
$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

The high-energy branch can be lost due to the shunt damping effect of the electrical load (*weak nonlinearity*).

Here We Examine Using a Bistable Piezomagnetoelastic Beam



(Moon and Holmes, 1979)



Magnets added near the tip of a cantilever introduce nonlinearity

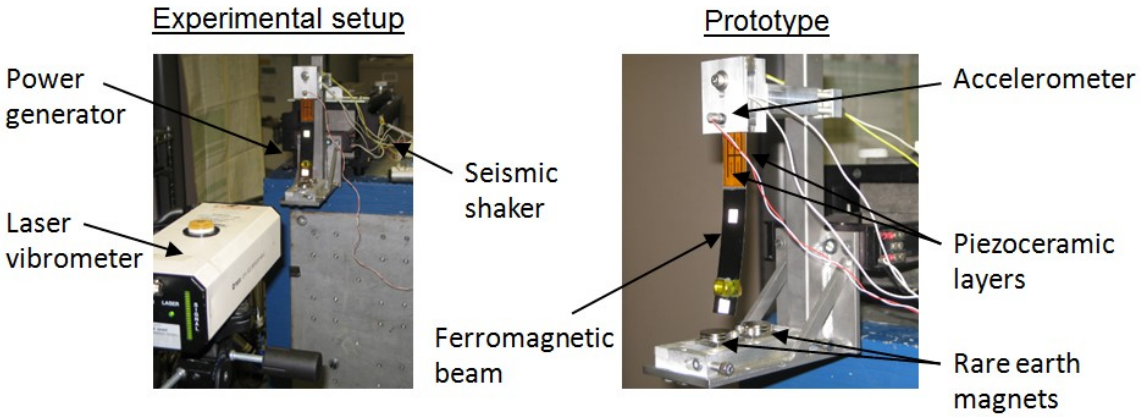
Limit Cycle Oscillations for Broad Band Harvesting

- A magnetic field causes the equation of motion of the harvesting piezoelectric cantilever to be nonlinear

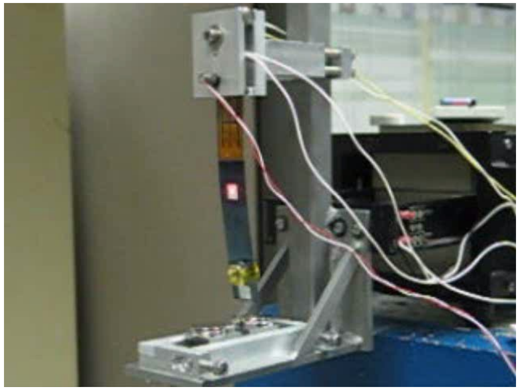
$$\ddot{x} + 2\zeta\dot{x} - \frac{1}{2}x\left(1 - x^2\right) - \chi v = f\cos\Omega t$$

- Spacing of the magnets results in: $\dot{v} + \lambda v + \kappa\dot{x} = 0$
  - 5 equilibrium (3 stable)
  - 3 equilibrium (2 stable)
  - 1 equilibrium (1 sable)
- Limit cycle oscillation is the possible producing large amplitude periodic response over a range of input frequencies

The piezomagnetoelastic energy harvester configuration has been investigated theoretically and experimentally.



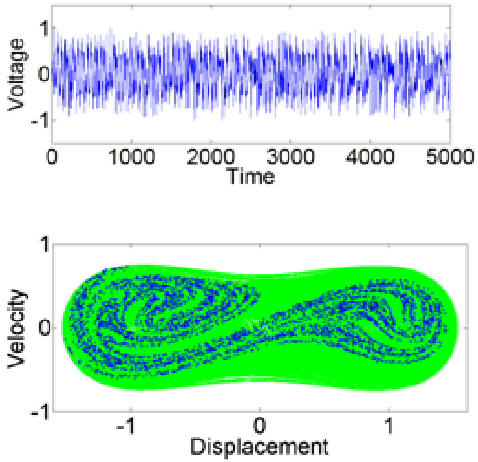
0.35g (RMS) at 8 Hz



[movie]

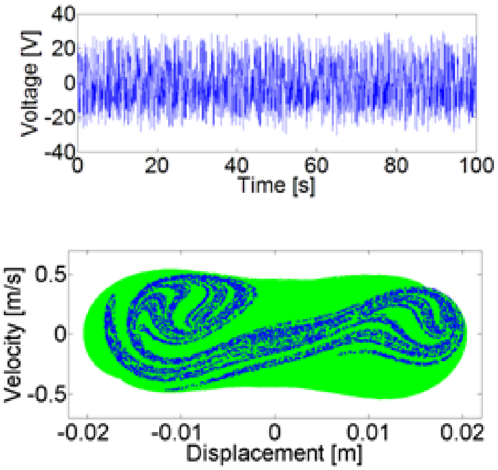
First the *strange attractor* (Moon and Holmes, 1979) is captured in the chaotic response of the piezomagnetoelastic configuration.

Theory



$\Omega = 0.8$

Experiment

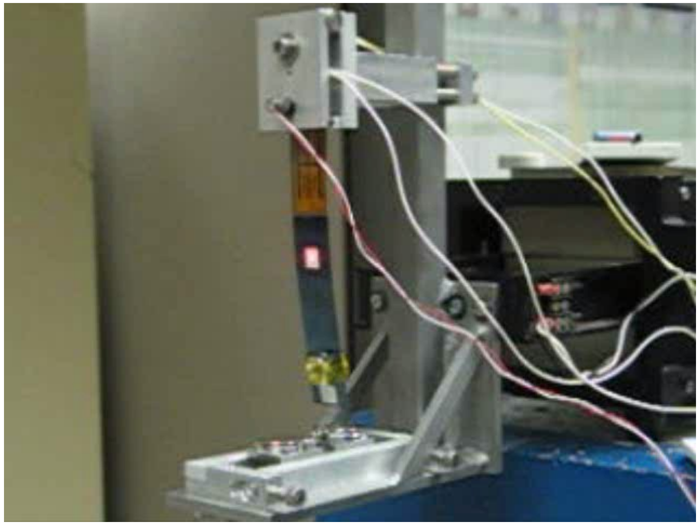


8 Hz

Large-amplitude periodic response is obtained by changing the forcing level or the initial conditions.

(1) Transient chaos followed by high-energy limit cycle oscillations (large-amplitude periodic attractor)

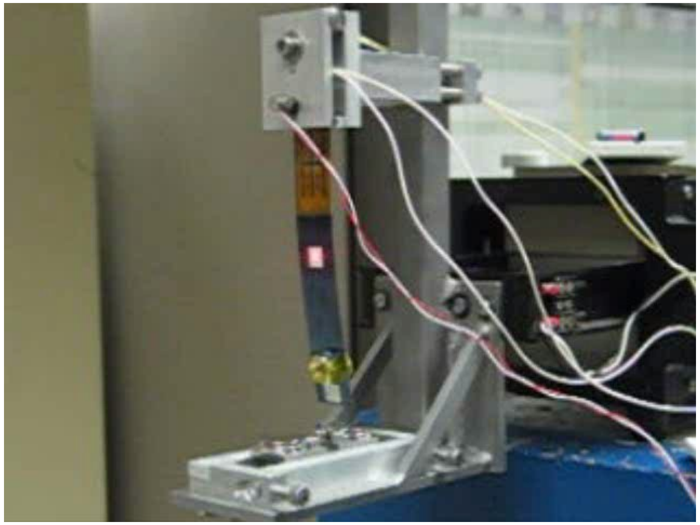
0.57*g* (RMS) input at 8 Hz



[movie]

(2) Co-existing attractors (strange attractor and large-amplitude periodic attractor)

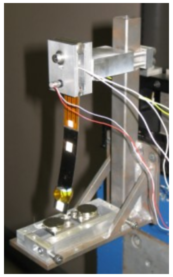
0.35*g* (RMS) input at 8 Hz



[movie]

Erturk, A., Hoffmann, J., and Inman, D.J., 2009, A Piezomagnetoelastic Structure for Broadband Vibration Energy Harvesting, *Applied Physics Letters*, **94**, 254102.

Theoretical simulations show the presence of these high-energy orbits at several frequencies.



Piezomagnetoelastic cantilever

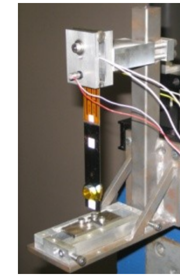
$$\ddot{x} + 2\zeta\dot{x} - \frac{1}{2}x\left(1 - x^2\right) - \chi v = f\cos\Omega t$$

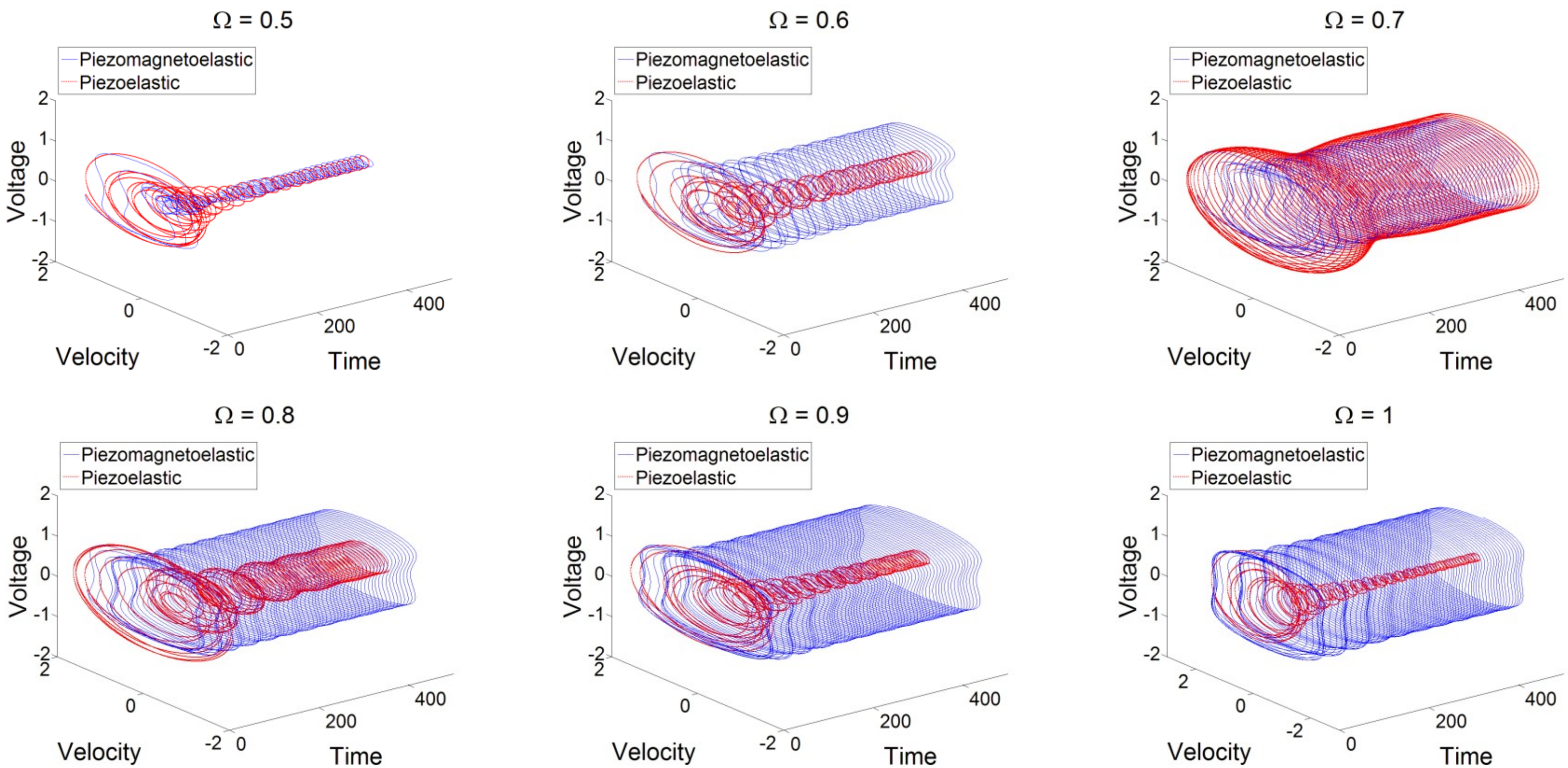
$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$

Piezoelastic cantilever

$$\ddot{x} + 2\zeta\dot{x} + \frac{1}{2}x - \chi v = f\cos\Omega t$$

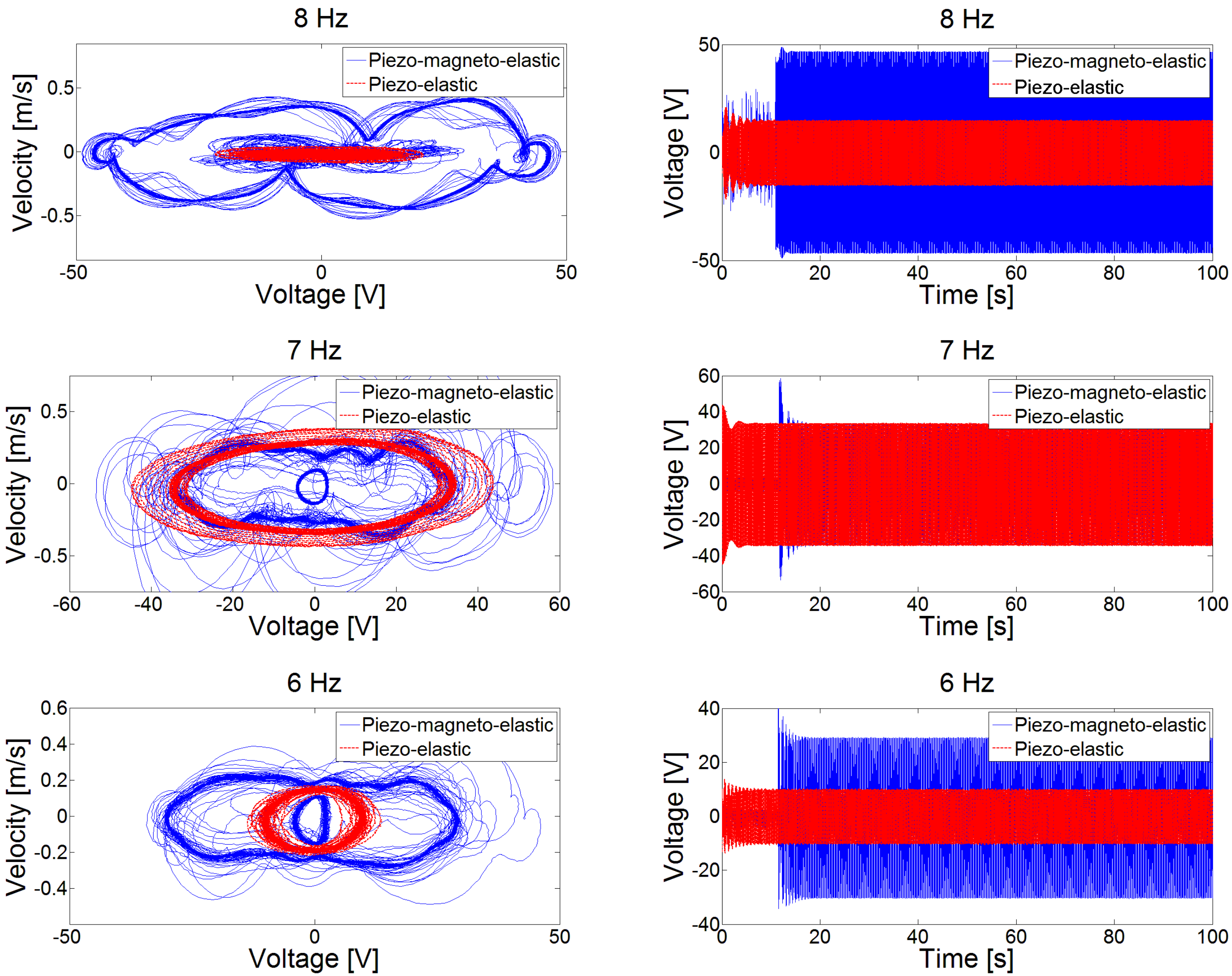
$$\dot{v} + \lambda v + \kappa\dot{x} = 0$$



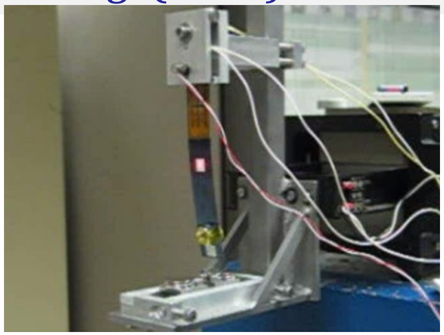


$\zeta = 0.01$, $\chi = 0.05$, $\kappa = 0.5$ and $\lambda = 0.05$ $x(0) = 1$, $\dot{x}(0) = 1.3$, $v(0) = 0$, $f = 0.08$

Experimental verification of the broadband high-energy orbits in the piezomagnetoelastic configuration

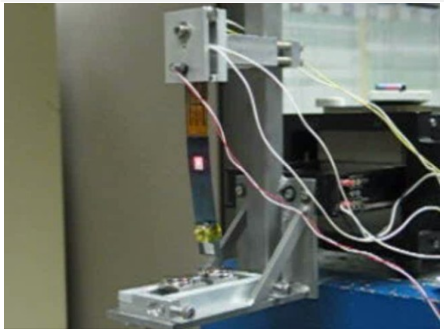


0.35*g* (RMS) at 8 Hz



[movie]

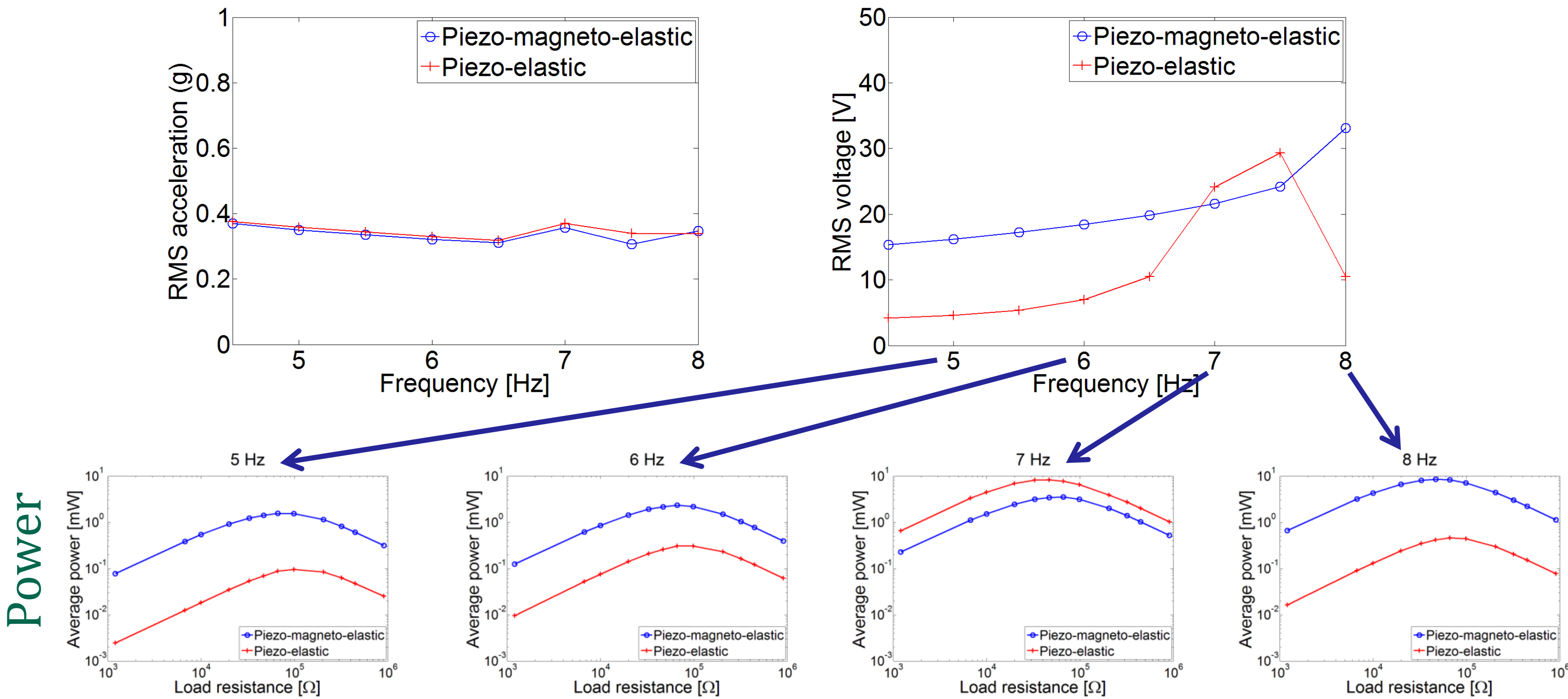
0.35*g* (RMS) at 6 Hz



[movie]

Large-amplitude response of the piezomagnetoelastic energy harvester yields an order of magnitude larger power output over a range of frequencies.

Base acceleration (input) *Open-circuit voltage* (output)



Power Output Comparison of Linear vs Nonlinear

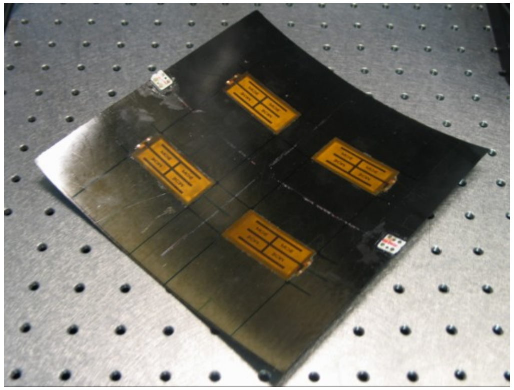
Excitation Frequency	**5 Hz**	**6 Hz**	**7 Hz**	**8 Hz**
Piezo-Magneto-Elastic	1.57 mW	2.33 mW	3.54 mW	8.54 mW
Piezo-elastic	0.10 mW	0.31 mW	8.23 mW	0.46 mW

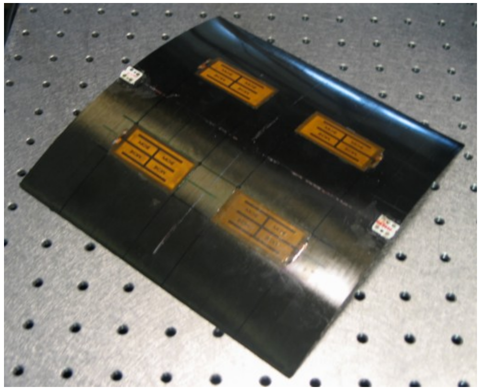
↑

Linear Resonance

Note that *at linear resonance* the linear system will always win, however it is narrow band and falls off quickly away from resonance and that the nonlinear has higher values overall

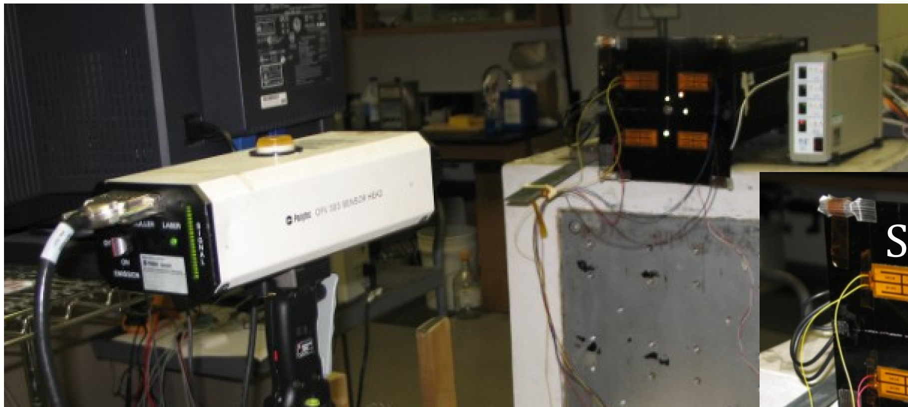
Bistable piezo-carbon-fiber-epoxy plate



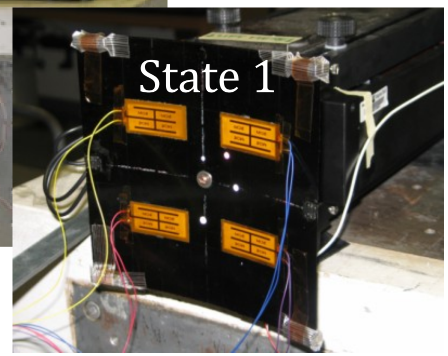


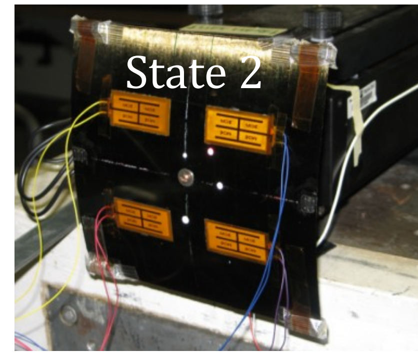
Courtesy of the Bristol Composites Group

A bistable carbon-fiber-epoxy plate exhibits similar nonlinear dynamics (no external magnets required).

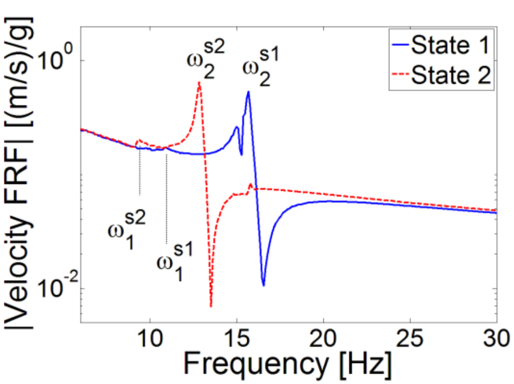


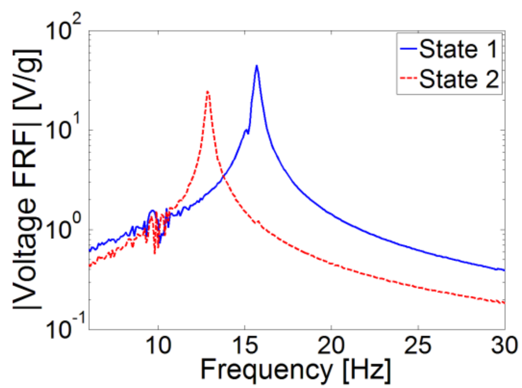
The plate is clamped to a seismic shaker from its center point.





Linear FRFs around each stable state



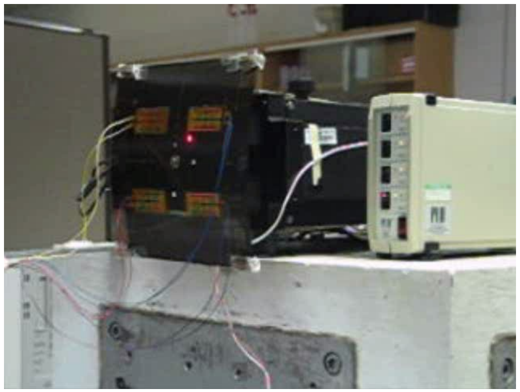
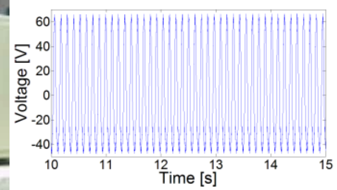


The stable equilibrium positions are not symmetric with respect to the unstable one.

Arrieta, A.F., Hagedorn, P., Erturk, A., and Inman, D.J., 2010, A Piezoelectric Bistable Plate for Nonlinear Broadband Energy Harvesting, *Applied Physics Letters*, **97**, 104102.

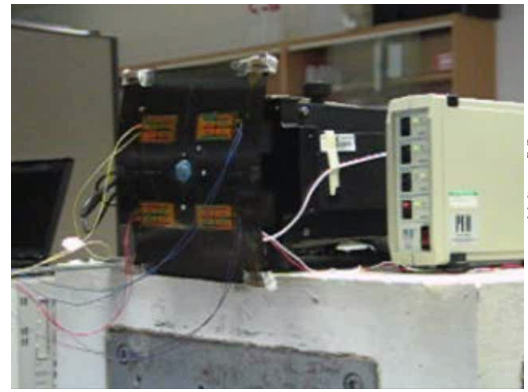
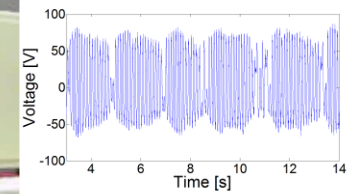
Various nonlinear phenomena can be observed in the bistable plate configuration.

High-energy LCO (8.6 Hz)

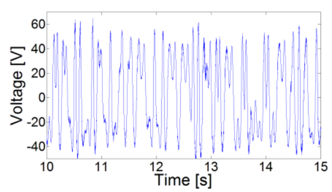
[movie]

Intermittent chaos (9.8 Hz)

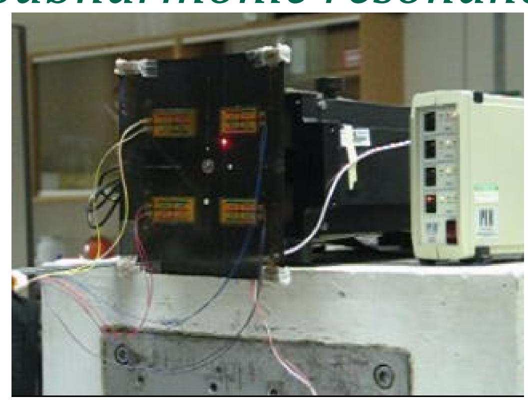
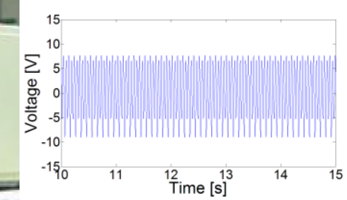
[movie]

Chaos (12.5 Hz)

[movie]

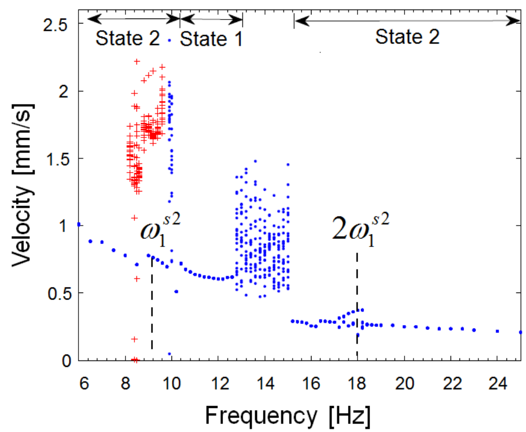
Subharmonic resonance (20.2 Hz)

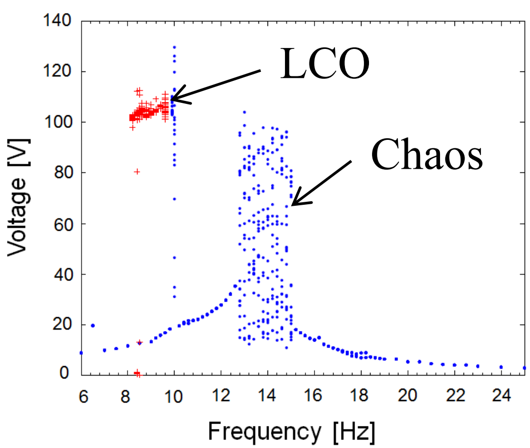



[movie]

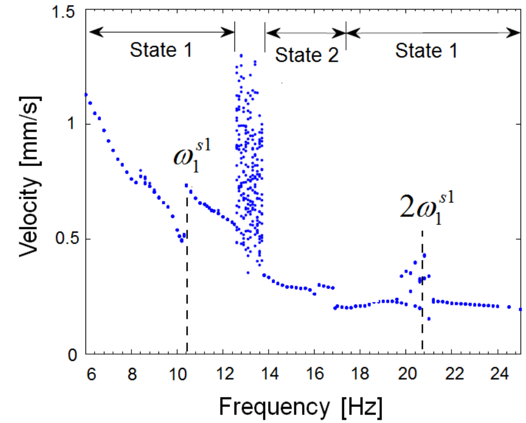
Various nonlinear phenomena can be observed in the bistable plate configuration.

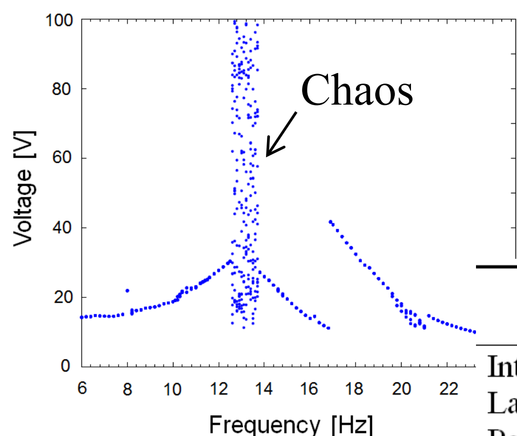
Forward sweep



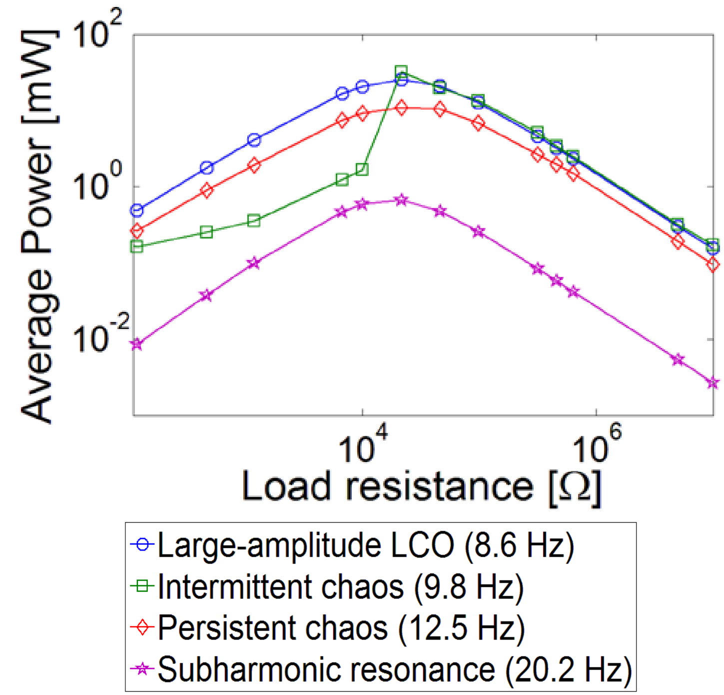


Backward sweep





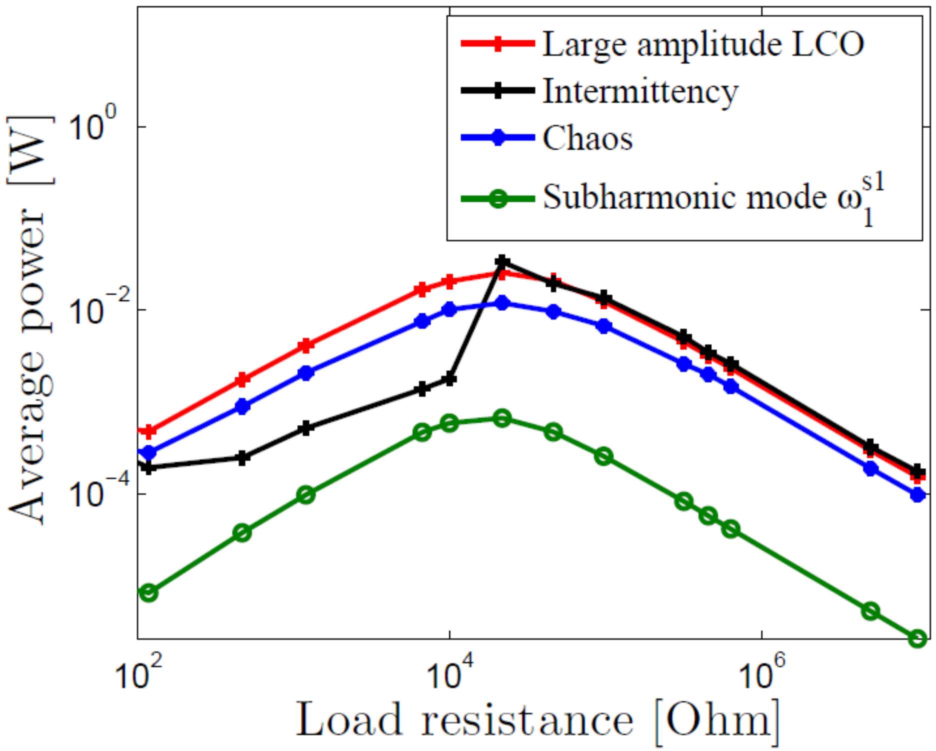
Power curves



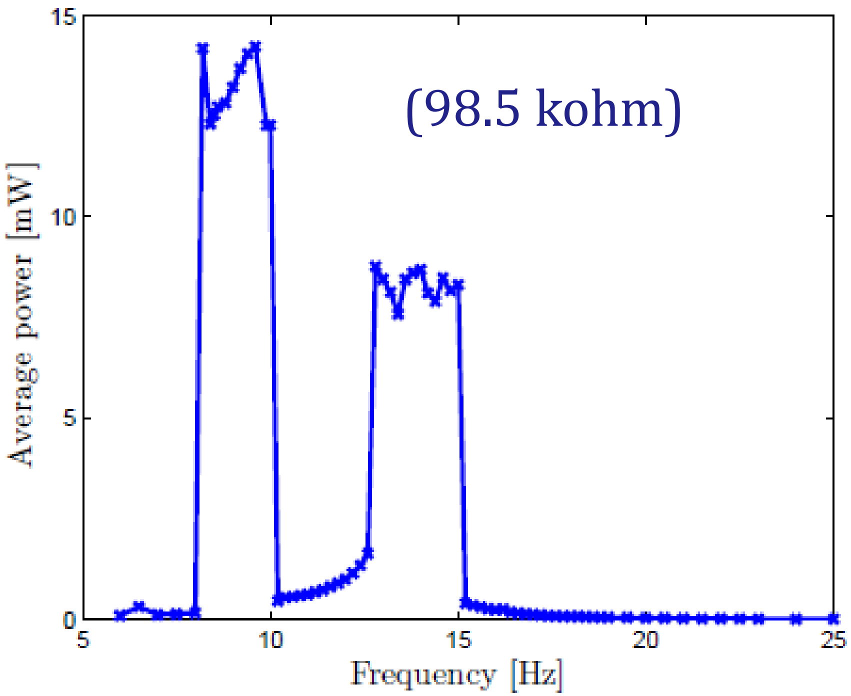
	Average power [mW]	Excitation frequency [Hz]
Intermittent chaos	34	9.8
Large-amplitude LCO	27	8.6
Persistent chaos	11	12.5
Subharmonic resonance	0.67	20.2

Large-amplitude oscillations generate very high power output over **a range of frequencies.**

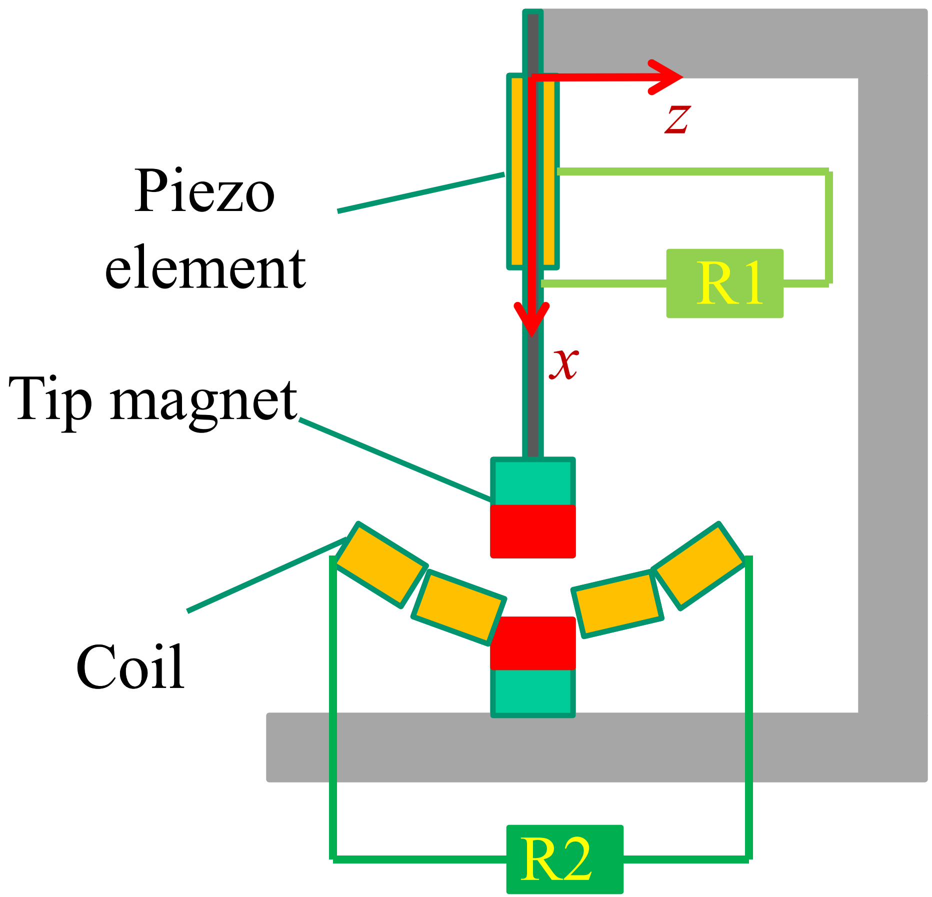
Average power vs. Load resistance



Average power vs. Frequency



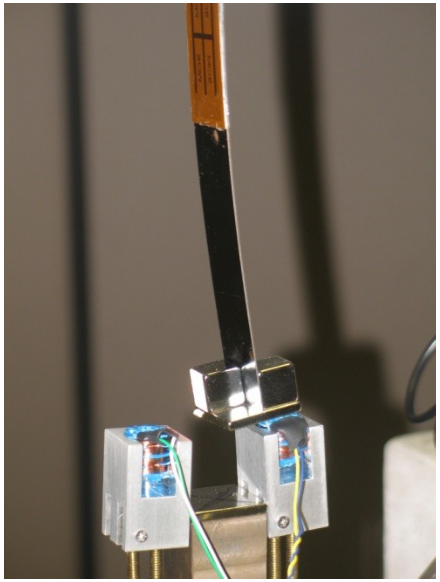
Nonlinear Hybrid Harvester

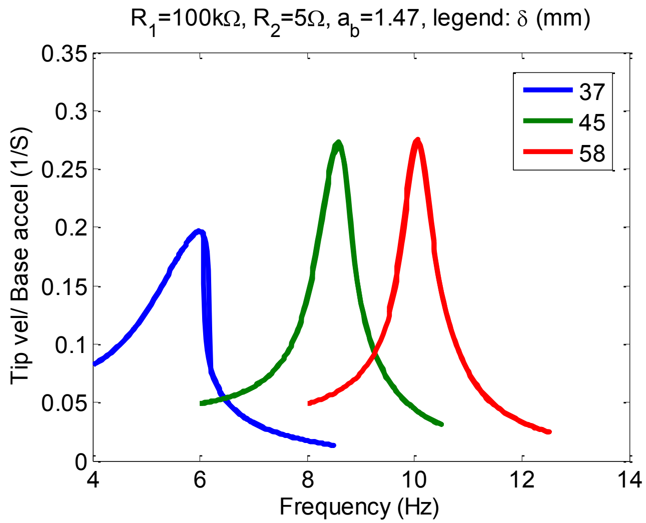


Piezoelectric Harvesting:
High voltage low current

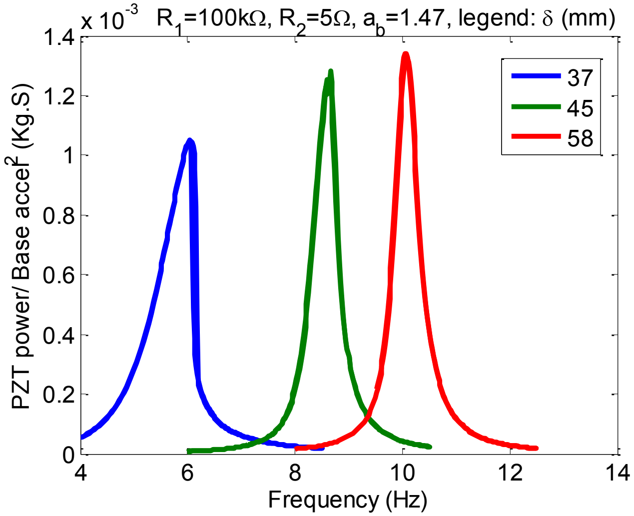
Electromagnetic Harvesting
High current low voltage

Mono-Stable : Magnet Spacing

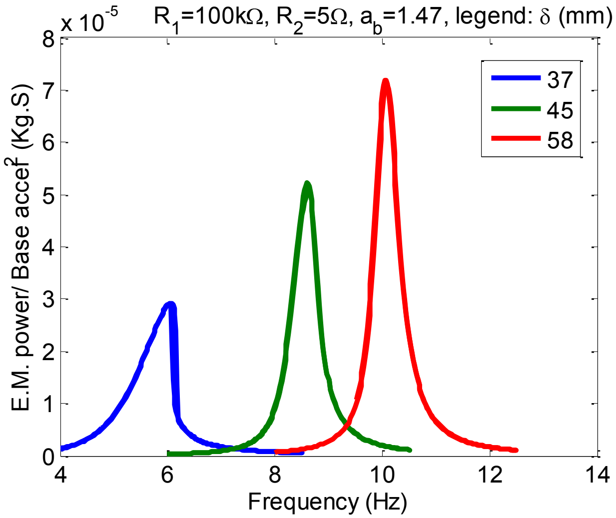




Tip Velocity

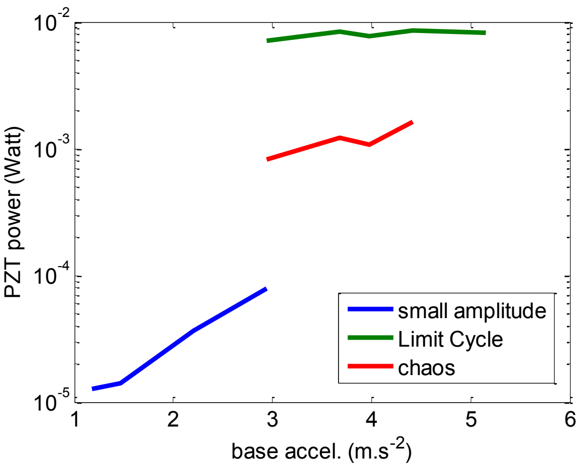


PZT Power

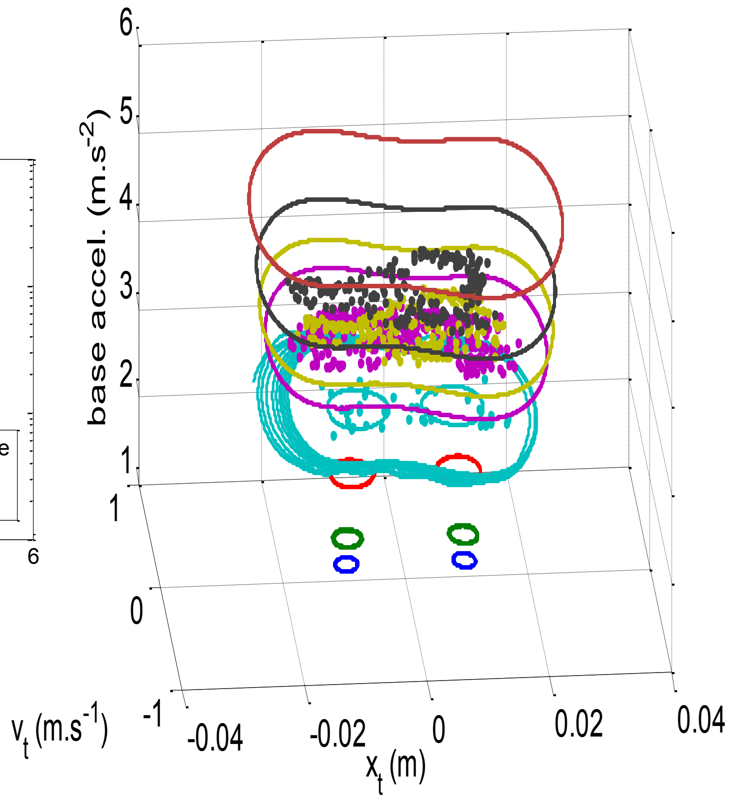


Electro/Mag Power

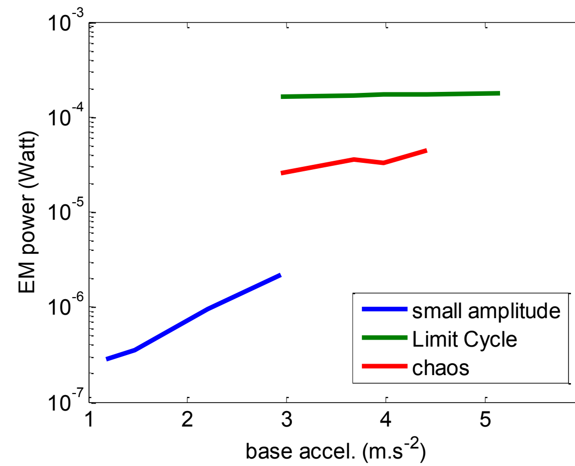
Bi-Stable : Base Acceleration



PZT Power

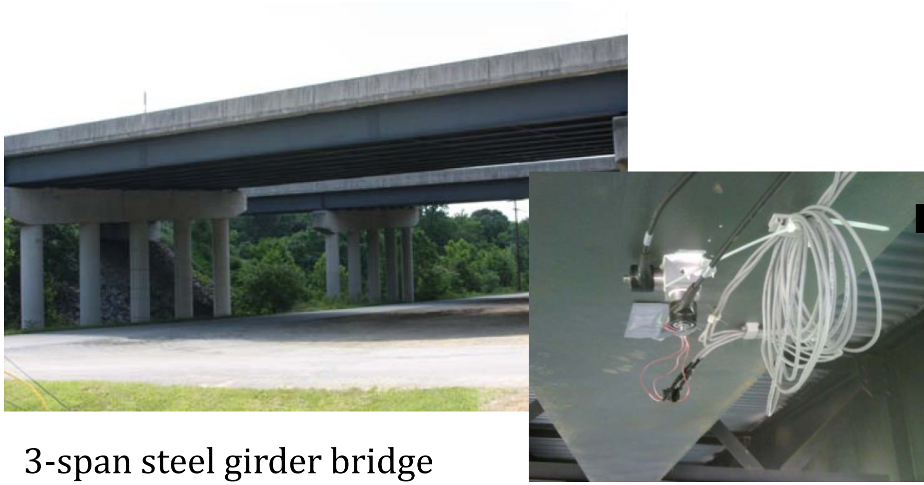


Tip Velocity



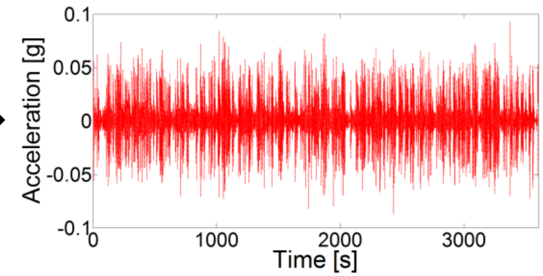
Electro/Mag Power

Acceleration data of the bridge has been simplified to a harmonic function for simulations in the lab.

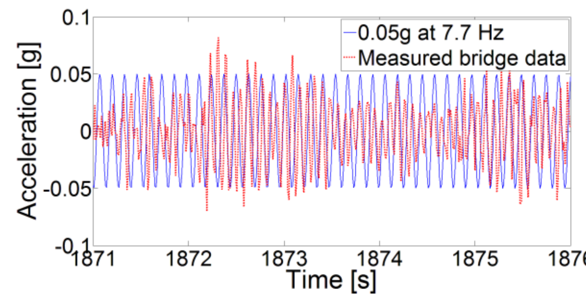


3-span steel girder bridge
(08/18/09 - Roanoke)

Acceleration signal measured on the bridge



Approximation as a persistent single harmonic (0.05*g* at 7.7 Hz)

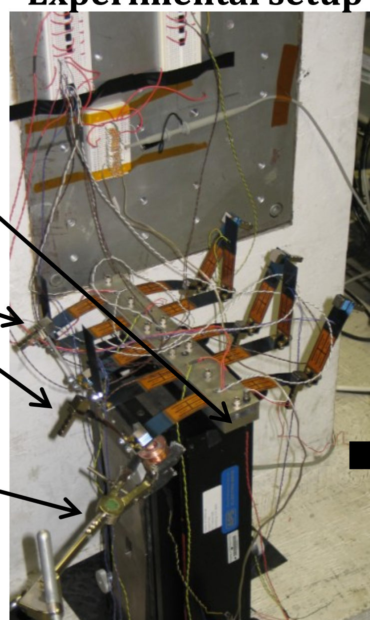


Experimental setup

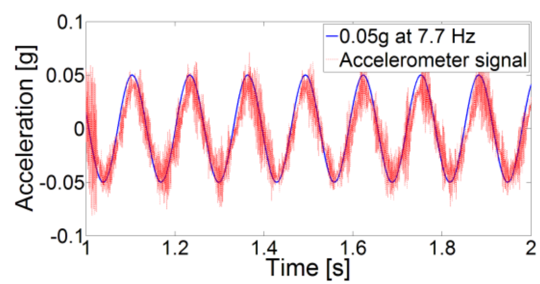
Accelerometer

Piezoelectric and electromagnetic generators

Seismic shaker

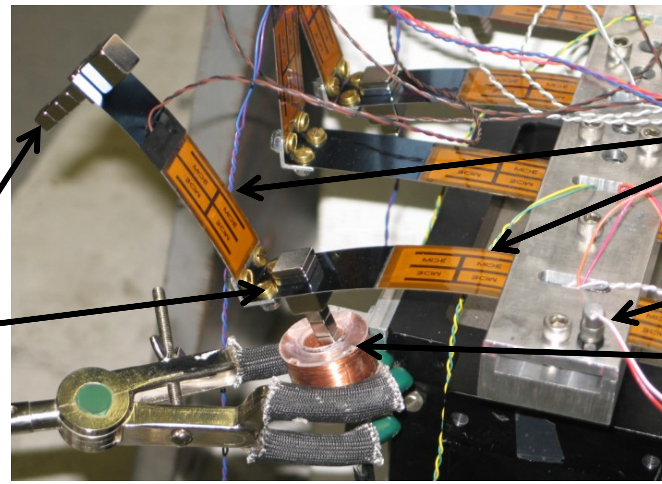


Acceleration measured on the shaker



Piezoelectric and electromagnetic power outputs have been measured for an acceleration input of 0.05g (RMS: 0.035g) at 7.7 Hz.

Combined piezoelectric-electromagnetic generator configuration



Electromagnetic part : 0.22 V for 82 ohms = **0.6 mW** (per coil)

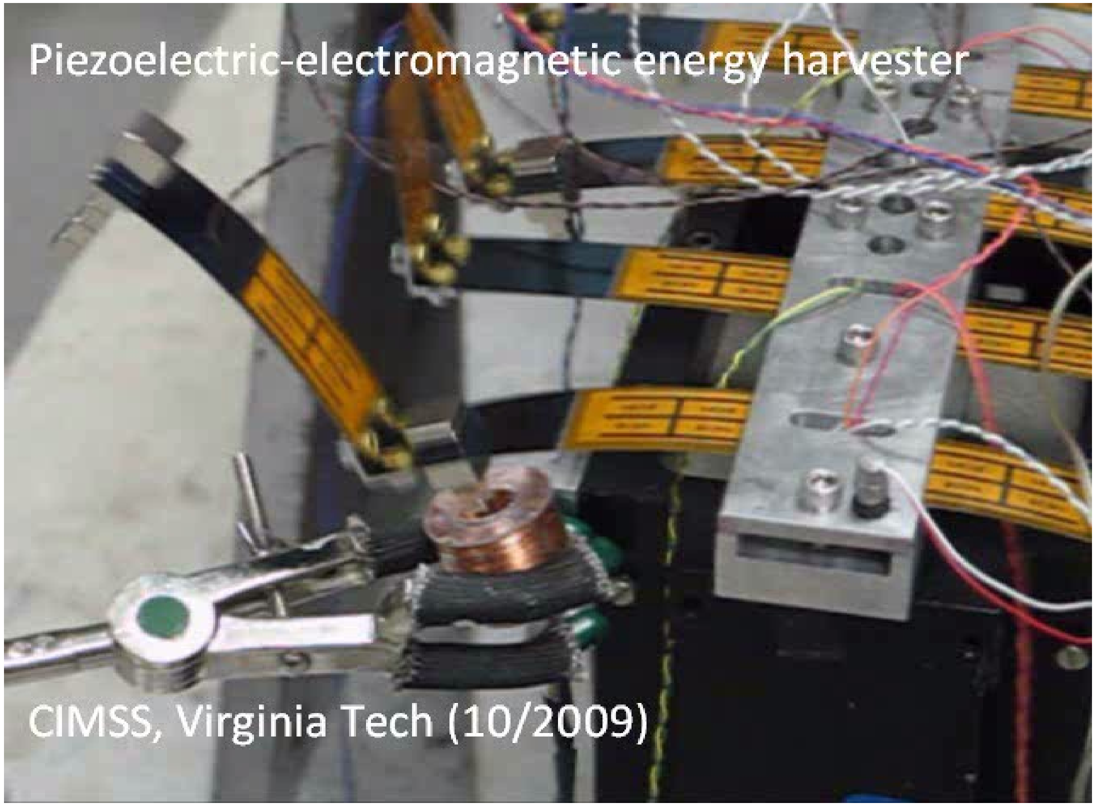
Piezoelectric part : 11.2 V for 470 kohms = **0.3 mW**

Power output of a single generator (for 0.05g) = 0.9 mW

Increased base acceleration amplitude results in a larger power output. (0.1*g*, RMS: 0.07*g* at 7.7 Hz yields 2.7 mW).

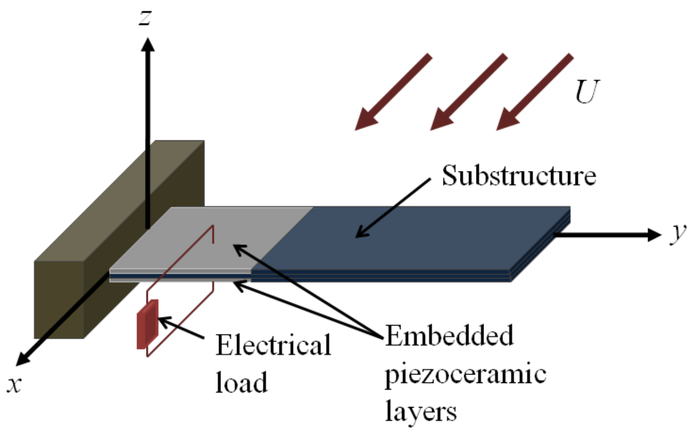
Electromagnetic part : 0.42 V for 100ohms = **1.8 mW** (from a single coil)
Piezoelectric part : 21 V for 470 kohms = **0.94 mW**

Power output of a single generator (for 0.1*g*) = 2.7 mW



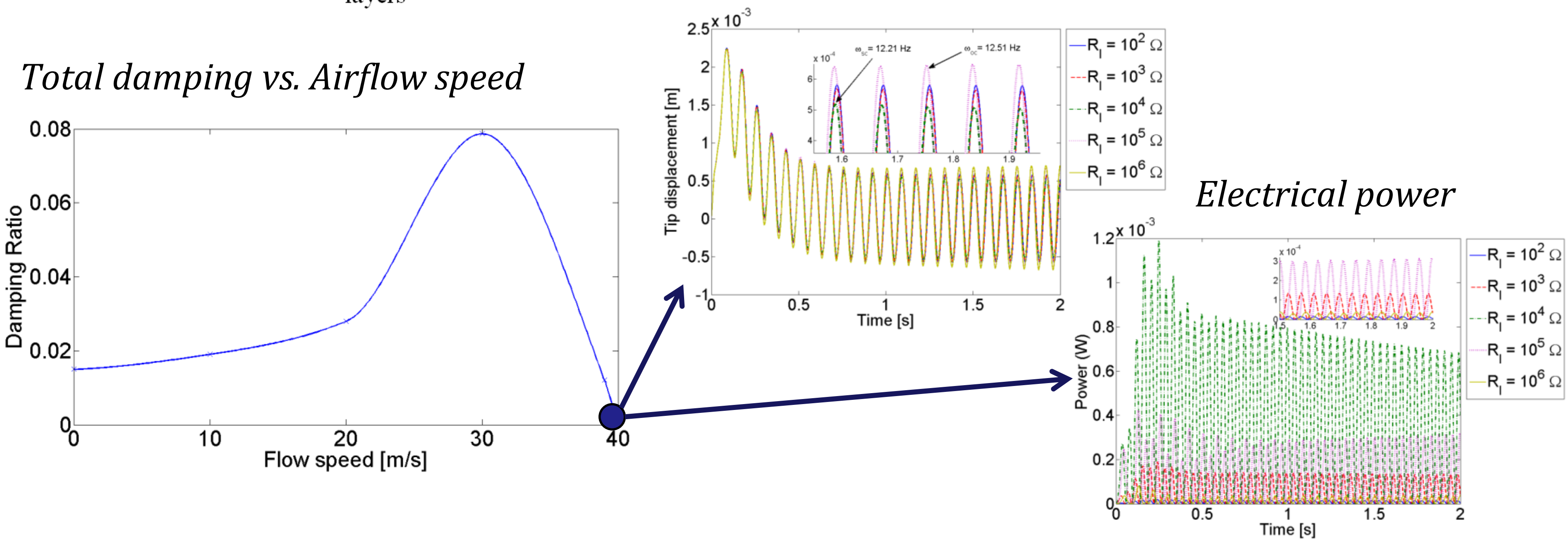
[click on the movie]

Is harvesting of flow through wing vibration possible?



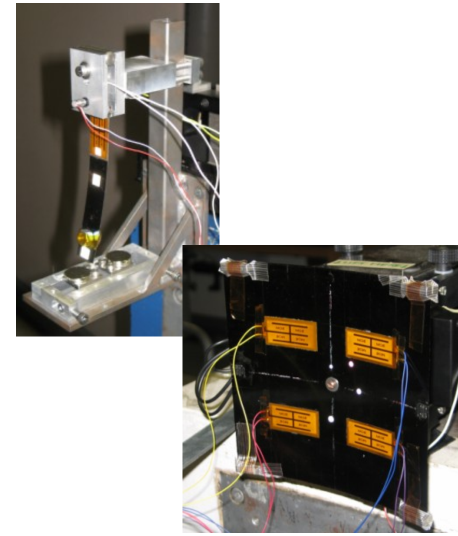
At 9.23 m/s, 10.7 mW harvested
AND
The corresponding shunt effect increased the flutter speed by 5.5%

Tip displacement

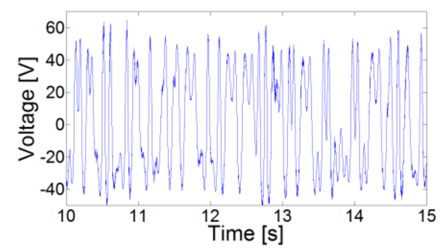


Just a linear analysis for now but LCO does occur in aircraft

Summary and conclusions



- Bistable beam and plate configurations have been discussed for broadband energy harvesting.
- The beam configuration requires magnets for bistability whereas the plate configuration is bistable due to the laminate characteristics.
- The design problem is to achieve persistent snap through and nonlinear phenomena for the given excitation amplitude and frequency range.



- Combined E/M and PZT is promising for charging batteries

Some Funded Projects In the US

- Center for Energy Harvesting Materials and Systems (National Science Foundation/Industry program)
  - ITT: new lead free piezoelectric materials
  - UTRC: building applications (running infrastructure sensors
  - SAIC: submerged river sensors (flow harvesting)
  - Texas Instruments: TBA
  - Physical Acoustics Corporation: Harvesting for running AE sensors
  - Texas MicroPower: MEMs Zigzag harvester
- National Institute for Standards and Testing
  - 50 million USD in harvesting and monitoring of Bridges
- Air Force Office of Scientific Research
  - 6 million USD for harvesting in UAVs

Acknowledgements

Air Force Office of Scientific Research grants monitored by Dr. "Les" B. L. Lee

- F 9550-06-1-0326: "*Energy Harvesting and Storage Systems for Future Air Force Vehicles*"

- F 9550-09-1-0625: "*Simultaneous Vibration Suppression and Energy Harvesting*"

Shameless, Self Serving References

Priya, S. and Inman, D. J., Editors, 2008, *Energy Harvesting Technologies*, Springer Science+Business Media, Inc., Norwell, MA, 517 pp.

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National Science Foundation Center for Energy Harvesting Materials and Systems

http://cehms.mse.vt.edu

Thank you!.. Questions?